MODELING OF TURBULENCE TRANSPORTING OF ADMIXTURE OVER HEATED SURFACE

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Представлена модель турбулентного течения, основанная на уравнениях вторых моментов для полей скорости, температуры и концентрации. Модель построена на аппроксимации локально равновесной турбулентности. Решение полученного уравнения представлено совокупностью двух факторов. Первый описывает однородное течение, а второй учитывает влияние силы Архимеда, которая зависит от числа Ричардсона. Модель использует минимальное количество эмпирических постоянных теории однородной турбулентности.

Introduction

The emissions into the moving medium in three-dimensional domain are one of actual problems. If the emissions enter into the moving medium, for example into the atmosphere, then usually the medium itself has already the turbulence due to the turbulence generation in the ground area of basic flow. In the domains, which are in a distance from an emission source, this turbulence dominates over the turbulence generated by an emission source and defines the distribution of polluting substances. The problems in such formulation are usually reduced to the investigation of passive admixture transference, where the admixture does not influence the basic moving medium.

The problem formulated in this paper, on the admixture, which is moving together with basic flow above a surface with non-uniform temperature and interacting with basic motion and temperature, has a great practical usage. In this case it is impossible to consider the admixture to be passive due to the complex correlations of velocity, temperature and admixture concentration.

Currently a range of the mathematical models has been created for the description of admixture dispersion processes. These models are based on the equations of turbulent diffusion and generally constructed for the velocity and one of the scalar value: either temperature or concentration. In the present work the mutual correlations of temperature, concentration and flow velocities are taken into account.

The volume forces have a great influence on the character of turbulence if they are interconnected with the pulsations of velocity. The simplest example is the strong influence of gravity on the flow with density pulsations. If density pulsations appear as the result of the existence of average density gradient in the same direction as average velocity gradient or when the flow actually arises due to a difference of average density, then the good correlation is appeared between pulsations of density and velocity. Therefore, the influence of buoyancy forces may become very great. In case when the density increases in a vertical direction from below to upwards we have the unstable flow. The interrelation of density and velocity may lead to the transformation of potential energy into turbulent kinetic energy. On the contrary when the density decreases from below to upwards faster than it is necessary for the preservation of hydrostatic balance of fluid, then the available turbulent energy can be transformed into potential energy. It means that turbulent mixing tends to decrease a gradient of density and therefore to raise the center of gravity of fluid volume.

1. Equations

In the given problem the main goal is to obtain the expression for turbulent characteristics in the explicit form via average characteristics of the turbulent flow. We use the equations describing the changes of Reynolds turbulent stresses [1, 2]:

$$\frac{\partial}{\partial \tau}\overline{u_i u_j} + U_k \frac{\partial}{\partial x_k}\overline{u_i u_j} + \overline{u_j u_k}\frac{\partial U_i}{\partial x_k} + \overline{u_i u_k}\frac{\partial U_j}{\partial x_k} -$$

$$-\overline{\frac{p}{\rho}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right)} - \frac{\partial}{\partial x_k}\left[\nu \frac{\partial}{\partial x_i}\overline{u_i u_j} - \overline{u_i u_j u_k} - \overline{(\delta_{jk}u_i + \delta_{ik}u_j)\frac{p}{\rho}}\right] +$$

$$+ 2\nu \overline{\frac{\partial u_j}{\partial x_k}\frac{\partial u_i}{\partial x_k}} + F_{ui} = 0. \tag{1}$$

For the description of complex turbulent flows, which have a temperature and a concentration, we use the additional equations for the second order moments of temperature and concentration fields [3, 4]:

$$\frac{\partial \overline{u_i t}}{\partial \tau} + \bar{U}_k \frac{\partial \overline{u_i t}}{\partial x_k} + \overline{u_k t}\frac{\partial \bar{U}_i}{\partial x_k} + \overline{u_i u_k}\frac{\partial T}{\partial x_k} +$$

$$+\frac{\partial}{\partial x_k}\left(-\nu \frac{\partial \overline{u_i t}}{\partial x_k} + \overline{u_k u_i t} + \overline{\frac{p}{\rho} t}\right) + 2\nu \overline{\frac{\partial u_i}{\partial x_k}\frac{\partial t}{\partial x_k}} + F_{ti} = 0; \tag{2}$$

$$\frac{\partial \overline{u_i q}}{\partial \tau} + \bar{U}_k \frac{\partial \overline{u_i q}}{\partial x_k} + \overline{u_k q}\frac{\partial \bar{U}_i}{\partial x_k} + \overline{u_i u_k}\frac{\partial Q}{\partial x_k} +$$

$$+\frac{\partial}{\partial x_k}\left(-\nu \frac{\partial \overline{u_i q}}{\partial x_k} + \overline{u_k u_i q} + \overline{\frac{p}{\rho} q}\right) + 2\nu \overline{\frac{\partial u_i}{\partial x_k}\frac{\partial q}{\partial x_k}} + F_{qi} = 0; \tag{3}$$

$$\frac{\partial \overline{t}^2}{\partial \tau} + U_k \frac{\partial \overline{t}^2}{\partial x_k} + 2\,\overline{u_k t}\frac{\partial T}{\partial x_k} - \frac{\partial}{\partial x_k}\left[-a\frac{\partial \overline{t}^2}{\partial x_k} + \overline{u_k t^2}\right] + 2a\overline{\frac{\partial t}{\partial x_k}\frac{\partial t}{\partial x_k}} = 0; \tag{4}$$

$$\frac{\partial \overline{q}^2}{\partial \tau} + U_k \frac{\partial \overline{q}^2}{\partial x_k} + 2\,\overline{u_k q}\frac{\partial \overline{Q}}{\partial x_k} + \frac{\partial}{\partial x_k}\left[-d\frac{\partial \overline{q}^2}{\partial x_k} + \overline{u_k q^2}\right] + 2d\overline{\frac{\partial q}{\partial x_k}\frac{\partial q}{\partial x_k}} = 0; \tag{5}$$

$$\frac{\partial \overline{qt}}{\partial \tau} + \bar{U}_k \frac{\partial \overline{qt}}{\partial x_k} + \overline{u_k t}\frac{\partial \overline{Q}}{\partial x_k} + \overline{u_k q}\frac{\partial T}{\partial x_k} + \frac{\partial}{\partial x_k}\left(-\nu\frac{\partial \overline{qt}}{\partial x_k} + \overline{u_k qt}\right) + (a+d)\overline{\frac{\partial q}{\partial x_k}\frac{\partial t}{\partial x_k}} = 0, \tag{6}$$

where τ — time; p — pressure; U_i, u_i — components of average and pulsation velocities respectively to the axes x_i; T, t — average and pulsation temperatures; Q, q — average and pulsation characteristics of the concentration. In the flow of a general type there exist six components of the tensor of Reynolds stresses $u_i u_j$ due to the symmetry; three components of the correlation $u_i t$ type and three components of the correlation $u_i q$ type; two equations for t^2 and q^2 and the equation for the correlation of tq type. Hence, it is necessary to solve fifteen partial differential equations. It is obvious that the equations contain some new unknown variables except the average velocity and the second order moments. For defining some terms of the equation system we use the approximated semi empirical ratios. The expressions for the change of energy of various components of pulsations are expressed in the form [3]:

$$\overline{\frac{p}{\rho}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)} = -k\frac{\sqrt{E}}{l}\left(\overline{u_i u_j}-\frac{2}{3}\delta_{ij}E\right),$$

$$\overline{\frac{P}{\rho}\frac{\partial t}{\partial x_k}} = -k_t\frac{\sqrt{E}}{l}\overline{u_i t}, \quad \overline{\frac{P}{\rho}\frac{\partial q}{\partial x_k}} = -k_q\frac{\sqrt{E}}{l}\overline{u_i q}. \tag{7}$$

For the dissipation of pulsation energy and its analogues the following expressions are used:

$$2\nu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}} = \nu\, c_v\frac{\overline{u_i}^2}{l^2}+\frac{2}{3}c\,\delta_{ij}\frac{E^{3/2}}{l},$$

$$2\nu\,\overline{\frac{\partial t}{\partial x_k}\frac{\partial t}{\partial x_k}} = c_{v\,t}a\nu\,\frac{\overline{t^2}}{l^2}+c_t\frac{\sqrt{E}}{l}\overline{t}^2, \tag{8}$$

$$2\nu\,\overline{\frac{\partial q}{\partial x_k}\frac{\partial q}{\partial x_k}} = c_{v\,t}\,\nu\,\frac{\overline{q^2}}{l^2}+c_q\frac{\sqrt{E}}{l}\overline{q}^2,$$

but for the second order moments they have the following form:

$$2\nu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}} = \nu\, c_v\frac{\overline{u_i u_j}}{l^2}, \quad 2\nu\,\overline{\frac{\partial t}{\partial x_k}\frac{\partial t}{\partial x_k}} = c_{v\,t}a\nu\,\frac{\overline{u_i t}}{l^2}, \quad 2\nu\,\overline{\frac{\partial q}{\partial x_k}\frac{\partial q}{\partial x_k}} = c_{v\,t}\,\nu\,\frac{\overline{u_i q}}{l^2}.$$

It is assumed that shear turbulent flows are considered in Boussinesq approximation, i. e. the changes of density are small and they are taken into account only in mass forces [4]:

$$F_{ui} = -\beta g\left(\delta_{3i}\overline{tu_j}+\delta_{3j}\overline{tu_i}\right)+\alpha g\left(\delta_{3i}\overline{qu_j}+\delta_{3j}\overline{qu_i}\right),$$

$$F_{ti} = g\delta_{3i}\left(-\beta\overline{t^2}+\alpha\overline{tq}\right), \quad F_{qi} = g\delta_{3i}\left(-\beta\overline{tq}+\alpha\overline{q^2}\right). \tag{9}$$

By writing down the equations (1)–(6) for the pure shear developed turbulent flow, neglecting the turbulent diffusion and closing these equations by the semi empirical hypotheses (7)–(9) we get the following system of equations:

$$\overline{u_j u_k}\frac{\partial U_i}{\partial x_k} + \overline{u_i u_k}\frac{\partial U_j}{\partial x_k} + k\frac{\sqrt{E}}{l}\left(\overline{u_i u_j}-\frac{2}{3}\delta_{ij}E\right)+\frac{2}{3}c\,\delta_{ij}\frac{E^{3/2}}{l}-$$

$$-\beta g\left(\delta_{3i}\overline{tu_j}+\delta_{3j}\overline{tu_i}\right)+\alpha g\left(\delta_{3i}\overline{qu_j}+\delta_{3j}\overline{qu_i}\right)=0,$$

$$\overline{u_k t}\frac{\partial \bar{U}_i}{\partial x_k}+\overline{u_i u_k}\frac{\partial T}{\partial x_k}+k_t\frac{\sqrt{E}}{l}\overline{u_i t}+g\delta_{3i}\left(-\beta\overline{t^2}+\alpha\overline{tq}\right)=0,$$

$$\overline{u_k t}\frac{\partial T}{\partial x_k} + c_t \frac{t^2\sqrt{E}}{l} = 0, \tag{10}$$

$$\overline{u_k q}\frac{\partial \bar{U}_i}{\partial x_k} + \overline{u_i u_k}\frac{\partial Q}{\partial x_k} + k_q \frac{\sqrt{E}}{l}\overline{u_i q} + g\delta_{3i}\left(-\beta\overline{tq} + \alpha\overline{q^2}\right) = 0,$$

$$\overline{u_k q}\frac{\partial Q}{\partial x_k} + c_q \frac{q^2\sqrt{E}}{l} = 0,$$

$$\overline{u_k t}\frac{\partial \overline{Q}}{\partial x_k} + \overline{u_k q}\frac{\partial T}{\partial x_k} + c_q \frac{\sqrt{E}}{l}\overline{tq} = 0,$$

where E — kinetic turbulent energy; l — scale of turbulence.

The solution of the equation system (10) regarding to pulsation characteristics consists of two factors. The first one corresponds to the flow in homogenous environment, but the second one takes into account Archimede's forces caused by temperature and concentration fields:

$$\begin{gathered}
E = E_0\varphi, \quad \overline{u_1^2} = \left(\overline{u_1^2}\right)_0\Omega_1, \quad \overline{u_2^2} = \left(\overline{u_2^2}\right)_0\Omega_2, \quad \overline{u_3^2} = (u_3^2)_0\Omega_3, \\
\overline{u_1u_3} = \left(\overline{u_1u_3}\right)_0\Omega_4, \quad \overline{u_2u_3} = \overline{(u_2u_3)}_0\Omega_5, \quad \overline{u_1u_2} = \left(\overline{u_1u_2}\right)_0\Omega_6, \\
t\overline{u_3} = \left(t\overline{u_3}\right)_0\Omega_9, \quad t\overline{u_1} = \left(t\overline{u_1}\right)_0\Omega_7, \quad t\overline{u_2} = \left(t\overline{u_2}\right)_0\Omega_8, \quad \bar{t}^2 = \left(\bar{t}^2\right)_0\Omega_{10}, \\
\overline{qu_3} = \left(\overline{qu_3}\right)_0\Omega_{13}, \quad \overline{qu_1} = \left(\overline{qu_1}\right)_0\Omega_{11}, \quad \overline{qu_2} = \left(\overline{qu_2}\right)_0\Omega_{12}, \quad \bar{q}^2 = (\bar{q}^2)_0\Omega_{14}, \\
\overline{qt} = \left(\overline{qt}\right)_0\Omega_{15}.
\end{gathered} \tag{11}$$

Let us note that the parameters of averaged flows are known. The expressions for the homogenous medium have the following basic form:

$$\left(\overline{u_3^2}\right)_0 = \frac{2}{3}\left(1 - \frac{c}{k}\right)\frac{1}{c^{2/3}}l^2\left[\left(\frac{\partial U_1}{\partial x_3}\right)^2 + \left(\frac{\partial U_2}{\partial x_3}\right)^2\right],$$

$$\left(\overline{u_1^2}\right)_0 = \frac{2}{3}\frac{c}{k}\frac{1}{c^{2/3}}l^2\left[\left(\frac{k}{c} - 1\right)\left(\frac{\partial U_2}{\partial x_3}\right)^2 + \left(\frac{k}{c} + 2\right)\left(\frac{\partial U_1}{\partial x_3}\right)^2\right],$$

$$(-\overline{u_1u_3})_0 = l^2\sqrt{\left(\frac{\partial U_2}{\partial x_3}\right)^2 + \left(\frac{\partial U_1}{\partial x_3}\right)^2}\left(\frac{\partial U_1}{\partial x_3}\right), \quad \left(\overline{u_1t}\right)_0 = 2\frac{c^{1/3}}{k_t}\left(1 + \frac{k}{k_t}\right)l^2\left(\frac{\partial U_1}{\partial x_3}\right)\left(\frac{\partial T}{\partial x_3}\right),$$

$$\left(-\overline{u_3t}\right)_0 = \frac{k}{k_t}l^2\sqrt{\left(\frac{\partial U_2}{\partial x_3}\right)^2 + \left(\frac{\partial U_1}{\partial x_3}\right)^2}\left(\frac{\partial T}{\partial x_3}\right), \quad (\overline{u_1q})_0 = 2\frac{c^{1/3}}{k_q}\left(1 + \frac{k}{k_q}\right)l^2\left(\frac{\partial U_1}{\partial x_3}\right)\left(\frac{\partial Q}{\partial x_3}\right),$$

$$(-\overline{u_3q})_0 = \frac{k}{k_q}l^2\sqrt{\left(\frac{\partial U_2}{\partial x_3}\right)^2 + \left(\frac{\partial U_1}{\partial x_3}\right)^2}\left(\frac{\partial Q}{\partial x_3}\right), \quad E_0 = \frac{1}{c^{2/3}}l^2\left[\left(\frac{\partial U_1}{\partial x_3}\right)^2 + \left(\frac{\partial U_2}{\partial x_3}\right)^2\right].$$

The functions taking into account the influence of stratification on the turbulent flow have the following form:

$$\Psi = \varphi^2 + \varphi\left[\mathrm{Rt}\left(\frac{k}{c_t} + \frac{k}{c_s}\frac{\mathrm{Pr}}{\mathrm{Sc}} + 2\right) - \mathrm{Rq}\left(\frac{k}{c_q} + \frac{k}{c_s}\frac{\mathrm{Sc}}{\mathrm{Pr}} + 2\right)\right] + \mathrm{Rt}^2\frac{k}{c_s}\frac{\mathrm{Pr}}{\mathrm{Sc}}\left(\frac{k}{c_t} + 2\right) +$$
$$+\mathrm{Rq}^2\frac{\mathrm{Sc}}{\mathrm{Pr}}\frac{k}{c_s}\left(2 + \frac{k}{c_q}\right) - \mathrm{Rt}\cdot\mathrm{Rq}\left[\frac{k}{c_t}\frac{k}{c_q} + 2\cdot\frac{k}{c_s}\left(\frac{c_t}{c_q} + \frac{c_q}{c_t}\right)\right],$$

$$\Omega_3 = \frac{\varphi}{\Psi}\left\{\varphi^2 + \varphi\left[\mathrm{Rt}\left(\frac{k}{c_t} + \frac{k}{c_s}\frac{\mathrm{Pr}}{\mathrm{Sc}}\right) - \mathrm{Rq}\left(\frac{k}{c_q} + \frac{k}{c_s}\frac{\mathrm{Sc}}{\mathrm{Pr}}\right)\right] + \right.$$

$$\left. + \frac{k}{c_s}\left(\mathrm{Rt}^2\frac{k}{c_t}\frac{\mathrm{Pr}}{\mathrm{Sc}} + \mathrm{Rq}^2\frac{k}{c_q}\frac{\mathrm{Sc}}{\mathrm{Pr}}\right) - \mathrm{Rt}\cdot\mathrm{Rq}\frac{k}{c_t}\frac{k}{c_q}\right\},$$

$$\Omega_1 = \frac{\varphi}{\frac{2}{3}\left(2 + \frac{c}{k}\right)\Psi}\left\{\frac{2}{3}\left(2 + \frac{c}{k}\right)\varphi^2 + \varphi\cdot\mathrm{Rt}\left[\frac{2}{3}\left(2 + \frac{c}{k}\right)\cdot\left(\frac{k}{c_t} + \frac{k}{c_s}\frac{\mathrm{Pr}}{\mathrm{Sc}}\right) + 4\right] - \right.$$

$$-\varphi\cdot\mathrm{Rq}\left[\frac{2}{3}\left(2 + \frac{c}{k}\right)\cdot\left(\frac{k}{c_q} + \frac{k}{c_s}\frac{\mathrm{Sc}}{\mathrm{Pr}}\right) + 4\right] + \mathrm{Rt}^2\frac{k}{c_s}\frac{\mathrm{Pr}}{\mathrm{Sc}}\left[\frac{2}{3}\left(2 + \frac{c}{k}\right)\frac{k}{c_t} + 4\right] +$$

$$\left. + \mathrm{Rq}^2\frac{k}{c_s}\frac{\mathrm{Sc}}{\mathrm{Pr}}\left[\frac{2}{3}\left(2 + \frac{c}{k}\right)\frac{k}{c_q} + 4\right] - \mathrm{Rt}\cdot\mathrm{Rq}\left[4\frac{k}{c_s}\left(\frac{c_t}{c_q} + \frac{c_q}{c_t}\right) + \frac{2}{3}\left(2 + \frac{c}{k}\right)\frac{k}{c_t}\frac{k}{c_q}\right]\right\},$$

$$\Omega_2 = \Omega_1,$$

$$\Omega_4 = \frac{\varphi^{3/2}}{(\varphi + \mathrm{Rt} - \mathrm{Rq})\,\Psi}\left\{\varphi^2 + \varphi\left[\mathrm{Rt}\left(\frac{k}{c_t} + \frac{k}{c_s}\frac{\mathrm{Pr}}{\mathrm{Sc}} - \frac{1}{\mathrm{Pr}}\right) - \mathrm{Rq}\left(\frac{k}{c_q} + \frac{k}{c_s}\frac{\mathrm{Sc}}{\mathrm{Pr}} - \frac{1}{\mathrm{Sc}}\right)\right] + \right.$$

$$\left. + \mathrm{Rt}^2\frac{k}{c_s}\frac{1}{\mathrm{Sc}}\left(\frac{k}{c_t} - 1\right) + \mathrm{Rq}^2\frac{k}{c_s}\frac{1}{\mathrm{Pr}}\left(\frac{k}{c_q} - 1\right) - \mathrm{Rt}\cdot\mathrm{Rq}\left[\frac{k}{c_s}\left(\frac{c_t}{c_q}\frac{1}{\mathrm{Pr}} + \frac{c_q}{c_t}\frac{1}{\mathrm{Sc}}\right) - \frac{k}{c_t}\frac{k}{c_q}\right]\right\},$$

$$\Omega_5 = \Omega_4, \qquad \Omega_6 = \frac{\Omega_4}{\sqrt{\varphi}}, \quad \Omega_9 = \frac{1}{\Psi}\left\{\varphi^{3/2}\left[\varphi + \frac{k}{c_s}\left(\mathrm{Rt}\frac{\mathrm{Pr}}{\mathrm{Sc}} - \mathrm{Rq}\frac{c_t}{c_q}\right)\right]\right\},$$

$$\Omega_7 = \frac{\varphi}{(\varphi + \mathrm{Rt} - \mathrm{Rq})(1 + \mathrm{Pr})\,\Psi}\left\{\varphi^2(1 + \mathrm{Pr}) + \varphi\cdot\mathrm{Rt}\left[\frac{k_t}{c_t} + \frac{k}{c_s}\frac{\mathrm{Pr}}{\mathrm{Sc}}(1 + \mathrm{Pr})\right] - \right.$$

$$-\varphi\cdot\mathrm{Rq}\left(\frac{k_t}{c_q} + \frac{k_q}{c_s} + 1 - \frac{\mathrm{Pr}}{\mathrm{Sc}} + \frac{k}{c_s}\frac{c_t}{c_q}\right) + \mathrm{Rt}^2\frac{k}{c_s}\frac{k_t}{c_t}\frac{\mathrm{Pr}}{\mathrm{Sc}} + \mathrm{Rq}^2\frac{k}{c_s}\left(\frac{k_q}{c_q} + \frac{c_t}{c_q} - 1\right) -$$

$$\left. - \mathrm{Rt}\cdot\mathrm{Rq}\left[\frac{k}{c_s}\left(\frac{\mathrm{Pr}}{\mathrm{Sc}} - \frac{c_q}{c_t}\frac{1}{\mathrm{Sc}}\right) + \frac{k_t}{c_t}\frac{k}{c_q}\right]\right\},$$

$$\Omega_8 = \Omega_7, \quad \Omega_{10} = \frac{\varphi}{\Psi}\left[\varphi + \frac{k}{c_s}\left(\mathrm{Rt}\frac{\mathrm{Pr}}{\mathrm{Sc}} - \mathrm{Rq}\frac{c_t}{c_q}\right)\right],$$

$$\Omega_{13} = \frac{1}{\Psi}\left\{\varphi^{3/2}\left[\varphi + \frac{k}{c_s}\left(\mathrm{Rt}\frac{c_q}{c_t} - \mathrm{Rq}\frac{\mathrm{Sc}}{\mathrm{Pr}}\right)\right]\right\},$$

$$\Omega_{11} = \frac{\varphi}{(\varphi + \mathrm{Rt} - \mathrm{Rq})(1 + \mathrm{Sc})\,\Psi}\left\{\varphi^2(1 + \mathrm{Sc}) - \varphi\cdot\mathrm{Rq}\left[\frac{k_q}{c_q} + \frac{k}{c_s}\frac{\mathrm{Sc}}{\mathrm{Pr}}(1 + \mathrm{Sc})\right] + \right.$$

$$+\varphi\cdot\mathrm{Rt}\left(\frac{k_t}{c_s} + \frac{k_q}{c_t} + 1 - \frac{\mathrm{Sc}}{\mathrm{Pr}} + \frac{k}{c_s}\frac{c_q}{c_t}\right) + \mathrm{Rq}^2\frac{k}{c_s}\frac{k_q}{c_q}\frac{\mathrm{Sc}}{\mathrm{Pr}} + \mathrm{Rt}^2\frac{k}{c_s}\left(\frac{k_t}{c_t} + \frac{c_q}{c_t} - 1\right) -$$

$$\left. - \mathrm{Rt}\cdot\mathrm{Rq}\left[\frac{k}{c_s}\left(\frac{\mathrm{Sc}}{\mathrm{Pr}} - \frac{c_t}{c_q}\frac{1}{\mathrm{Pr}}\right) + \frac{k_q}{c_q}\frac{k}{c_t}\right]\right\},$$

$$\Omega_{12} = \Omega_{11}, \quad \Omega_{14} = \frac{\varphi}{\Psi}\left[\varphi + \frac{k}{c_s}\left(\mathrm{Rt}\frac{c_q}{c_t} - \mathrm{Rq}\frac{\mathrm{Sc}}{\mathrm{Pr}}\right)\right],$$

$$\Omega_{15} = \frac{\varphi}{(\mathrm{Sc} + \mathrm{Pr})\,\Psi}\left[\varphi\cdot(\mathrm{Sc} + \mathrm{Pr}) + \mathrm{Rt}\frac{k}{c_s}\mathrm{Pr}\left(\frac{c_q}{c_t} + 1\right) - \mathrm{Rq}\frac{k}{c_s}\mathrm{Sc}\left(\frac{c_t}{c_q} + 1\right)\right],$$

$$\varphi = \frac{1}{3}\left[1 - \mathrm{Rt}\left(\lambda_1 + \frac{k}{c_s}\frac{\mathrm{Pr}}{\mathrm{Sc}}\right) + \mathrm{Rq}\left(\lambda_2 + \frac{k}{c_s}\frac{\mathrm{Sc}}{\mathrm{Pr}}\right)\right] + \left(\sqrt{\Phi} - \Theta\right)^{1/3} - \left(\sqrt{\Phi} + \Theta\right)^{1/3},$$

$$\Theta = \left(\frac{\omega}{3}\right)^3 - \frac{\omega\zeta}{6} + \frac{\phi}{2}, \qquad \Phi = \Theta^2 + \left(\frac{\zeta}{3} - \frac{\omega^2}{9}\right)^3,$$

$$\lambda_1 = \frac{2}{3}\left(\frac{k}{c} - 1\right) + \frac{k}{c_t} + 3, \qquad \lambda_2 = \frac{2}{3}\left(\frac{k}{c} - 1\right) + \frac{k}{c_q} + 3,$$

$$\lambda_3 = \frac{k}{c_s}\frac{2}{3}\left(2 + \frac{k}{c}\right)\left(\frac{c_t}{c_q} + \frac{c_q}{c_t}\right) + \frac{k}{c_t}\frac{k}{c_q}, \quad \omega = \mathrm{Rt}\left(\lambda_1 + \frac{k}{c_s}\frac{\mathrm{Pr}}{\mathrm{Sc}}\right) - \mathrm{Rq}\left(\lambda_2 + \frac{k}{c_s}\frac{\mathrm{Sc}}{\mathrm{Pr}}\right) - 1,$$

$$\zeta = \mathrm{Rt}^2\left[\lambda_1\left(\frac{k}{c_s}\frac{\mathrm{Pr}}{\mathrm{Sc}} + 1\right) - 1\right] + \mathrm{Rq}^2\left[\lambda_2\left(\frac{k}{c_s}\frac{\mathrm{Sc}}{\mathrm{Pr}} + 1\right) - 1\right] -$$

$$-\mathrm{Rt}\cdot\mathrm{Rq}\cdot\left[\frac{k}{c_s}\left(\frac{\mathrm{Sc}}{\mathrm{Pr}} + \frac{\mathrm{Pr}}{\mathrm{Sc}}\right) - 2 + \lambda_1 + \lambda_2 + \lambda_3\right] +$$

$$+\mathrm{Rt}\left(\frac{1}{\mathrm{Pr}} - \frac{k}{c_s}\frac{\mathrm{Pr}}{\mathrm{Sc}} - \frac{k}{c_t}\right) - \mathrm{Rq}\left(\frac{1}{\mathrm{Sc}} - \frac{k}{c_s}\frac{\mathrm{Sc}}{\mathrm{Pr}} - \frac{k}{c_q}\right),$$

$$\phi = (\mathrm{Rt} - \mathrm{Rq})\left\{\left[\mathrm{Rt}^2\frac{k}{c_s}\frac{\mathrm{Pr}}{\mathrm{Sc}}(\lambda_1 - 1) + \mathrm{Rq}^2\frac{k}{c_s}\frac{\mathrm{Sc}}{\mathrm{Pr}}(\lambda_2 - 1)\right] - \mathrm{Rt}\cdot\mathrm{Rq}\cdot\lambda_3\right\} +$$

$$+\mathrm{Rt}^2\frac{k}{c_s}\frac{1}{\mathrm{Sc}}\left(1 - \frac{k_t}{c_t}\right) + \mathrm{Rq}^2\frac{k}{c_s}\frac{1}{\mathrm{Pr}}\left(1 - \frac{k_q}{c_q}\right) + \mathrm{Rt}\cdot\mathrm{Rq}\cdot\frac{k}{c_s}\left(\frac{k}{c_s} - \frac{c_t}{c_q}\frac{1}{\mathrm{Pr}} - \frac{c_q}{c_t}\frac{1}{\mathrm{Sc}}\right).$$

Let us note that some functions coincide due to the symmetry of the initial equations.

$$\mathrm{Rt} = \frac{2}{3}\frac{\beta g\dfrac{\partial T}{\partial x_3}}{\mathrm{Pr}\left(\dfrac{k}{c} - 1\right)\left[\left(\dfrac{\partial U_1}{\partial x_3}\right)^2 + \left(\dfrac{\partial U_2}{\partial x_3}\right)^2\right]}, \quad \mathrm{Rq} = \frac{2}{3}\frac{\alpha g\dfrac{\partial Q}{\partial x_3}}{\mathrm{Sc}\left(\dfrac{k}{c} - 1\right)\left[\left(\dfrac{\partial U_1}{\partial x_3}\right)^2 + \left(\dfrac{\partial U_2}{\partial x_3}\right)^2\right]},$$

where Rt, Rq — Richardson's numbers depending on temperature and concentration respectively. Pr, Sc — turbulent Prandtl's and Schmidt's numbers respectively depending on physical properties of a fluid. All constants c_q, c_t, c_s, k_t, k_q are determined via k and c, which can be found as follows:

$$k = \sqrt{\frac{c}{k}}\left[\frac{2}{3}\left(\frac{k}{c} - 1\right)\right]^{3/4}, \quad c = \left(\frac{c}{k}\right)^{3/2}\left[\frac{2}{3}\left(\frac{k}{c} - 1\right)\right]^{3/4},$$

where the constant $k/c = 7$ is determined from the theory of isotropic turbulence as the anisotropy coefficient independent of a type of a flow. Thus, the obtained expressions allow to close the Reynolds's equations for complex flows and to calculate turbulent pulsation characteristics of a flow.

2. Basic equations

In order to study the interaction of the fields of velocity, temperature and admixture concentration, the advanced turbulent flow will be considered in the three-dimensional aerodynamic channel.

The Reynolds three-dimensional non-stationary equations, turbulent heat and concentration transference are used:

$$\begin{gathered}
\frac{\partial U_i}{\partial \tau} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\langle -u_j u_i\rangle - \delta_{i3} g \rho', \\
\frac{\partial T}{\partial \tau} + U_j \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j}\left(-\overline{u_j t}\right), \\
\frac{\partial Q}{\partial \tau} + U_j \frac{\partial Q}{\partial x_j} = \frac{\partial}{\partial x_j}\left(-\overline{u_j q}\right), \\
\frac{\partial U_j}{\partial x_j} = 0, \\
\rho' = -\beta T + \alpha Q.
\end{gathered} \tag{12}$$

It is assumed that the resulting velocity U_g, which is parallel to the wall, locates on harsh borders at the point x^c just behind the viscous sublayer. The resulting velocity is expressed via the dynamic velocity by the logarithmic law of the wall:

$$\frac{U_g}{U_*} = \frac{1}{\kappa}\ln\left(y^c G\right),$$

where $y^c = x^c U_*/\nu$, $\kappa = 0,4$ is von Karman's constant, $G= 9$ is the coefficient of wall roughness, U_* is the dynamic velocity on a wall, x^c is such that the condition is fulfilled: $30 \le y^c \le 100$.

The following conditions are taken for the temperature and the concentration:

$$\frac{\partial T}{\partial x_2} = 0, \quad \frac{\partial Q}{\partial x_2} = 0$$

on the lateral walls,

$$T = T_s, \quad \frac{\partial Q}{\partial x_3} = 0 \tag{13}$$

on the lower wall,

$$T = T_b$$

on the heated surface,

$$\frac{\partial Q}{\partial x_3} = Q_s$$

on the source of concentration.

The following conditions are set on the inlet:

$$U_1 = U_0, \quad U_2 = 0, \quad U_3 = 0, \quad T = T_v, \quad \frac{\partial Q}{\partial x_1} = 0. \tag{14}$$

On the outlet:

$$\frac{\partial U_1}{\partial x_1} = 0, \quad U_2 = 0, \quad U_3 = 0, \quad \frac{\partial T}{\partial x_1} = 0, \quad \frac{\partial S}{\partial x_1} = 0. \tag{15}$$

The system of equations (12) is closed by the turbulence model (11) and solved by the numerical method [6] taking into account the boundary conditions (13)–(15).

3. Numerical Results

The stated model is applied to solve the problem described in the paper [5]. The movement of stratified air in rectangular three-dimensional domain is considered in the experiment. The height is $H_3 = 60\,\text{sm}$, the width is $H_2 = 182\,\text{sm}$ and the length is $H_1 = 360\,\text{sm}$. There is the linear source with the length $H_2 = 152\,\text{sm}$ in front of the heated square plate. This source is positioned across the flow. The admixture is entering from the source to the basic flow motion. The parameters of basic flow are taken as follows. The average velocity of the flow is $U_0 = 1,25\,\text{m/s}$, the temperature of the flow is $T_v = 43\,°\text{C}$, the temperature of the heated plate is $T_b = 121\,°\text{C}$, the temperature outside of the heated plate is $T_s = 4\,°\text{C}$.

The characteristics of the averaged fields of velocity, the spatial distribution of temperature and the transfer of concentration are obtained. Fig. 1 shows the profiles of longitudinal velocity U_1/U_m on different distances from the beginning of the heated surface. 1 is the profile of flow on the beginning of the heated plate, 2 is the profile on the middle of the heated plate, 3 is the profile on the end of the heated plate. Above the heated surface there is the deformation of the flow caused by thermal convection. The longitudinal velocity on the top of the channel is accelerated; and the velocity is decreased directly above the heated surface. Fig. 2 shows the velocity vector field in the cross section of the flow at the end of the heated plate, where secondary flows are formed. Fig. 3 presents the spatial distributions U_1/U_m at the same section. In fig. 3 we can see that the longitudinal component of velocity above the heated surface is decreased, but the velocity is increased on lateral edges of the channel. Fig. 4 shows contour distributions of concentration of the admixture in longitudinal section in the middle of the channel along the basic flow. In fig. 5 we can see contour distributions of the admixture concentration in the represented cross section. By comparing fig. 5 and fig. 2 it is can be seen that the velocity vector field and the concentration field are correlated. The distribution of temperature in the cross section at the end of the heated plate is shown in fig. 6. The modeling results coincide with the experimental data. They describe the basic laws of transfer of the admixture above the heated surface.

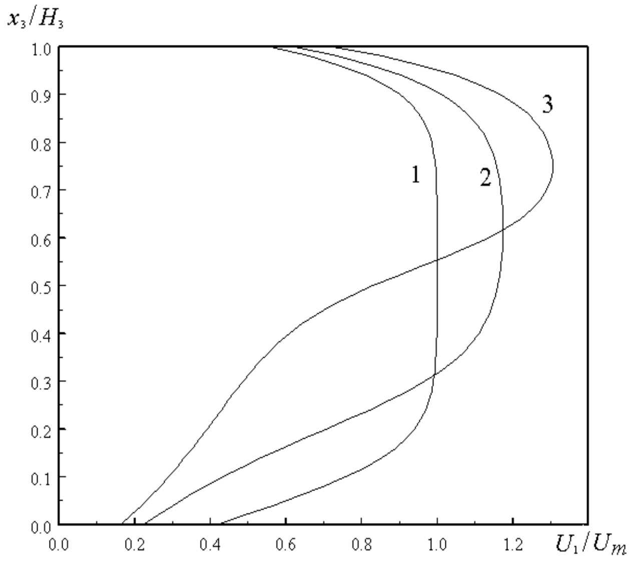

Fig. 1.

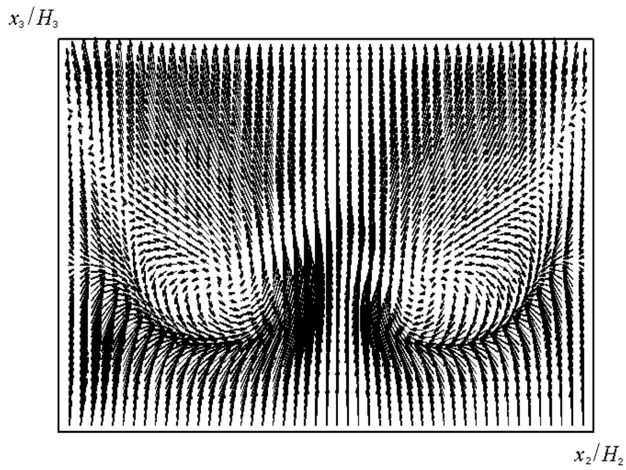

Fig. 2.

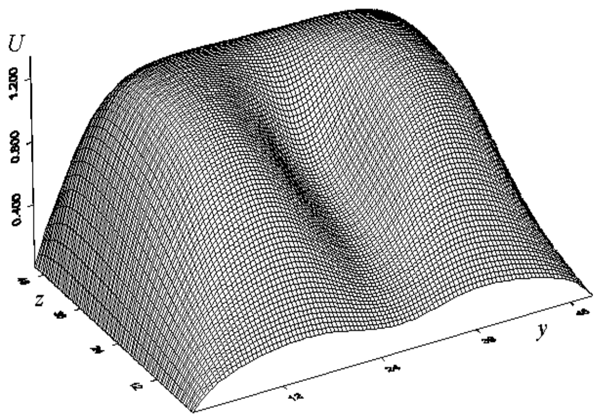

Fig. 3.

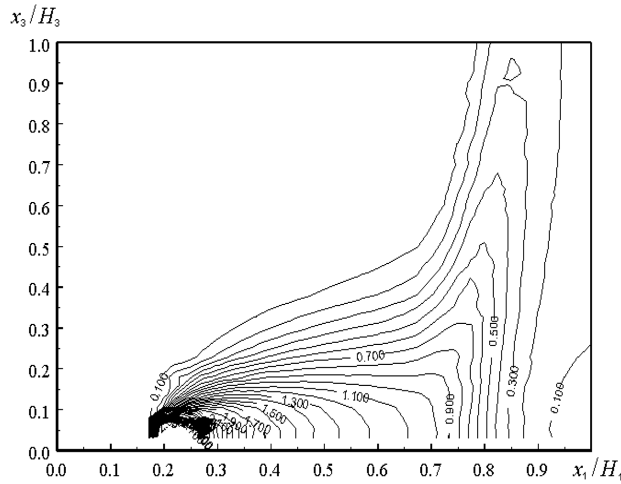

Fig. 4.

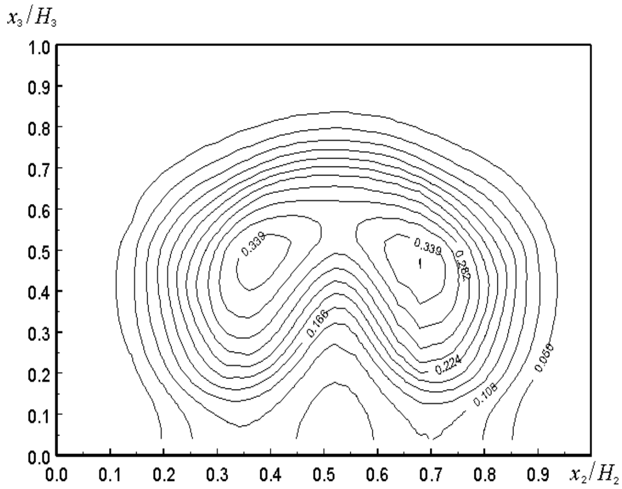

Fig. 5.

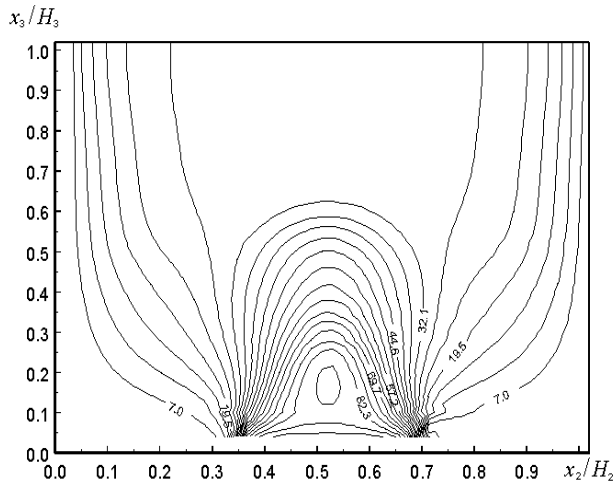

Fig. 6.

Thus, the turbulence model allows solving the problems on the transfer of admixture above the temperature-non-uniform spreading surface and to calculate the fields of average velocity, temperatures and concentration. Additionally the mutual correlations of these fields are taken into account. This turbulence model can be applied for modeling the turbulent transfer of admixture in stratified medium.

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