ONE ω-INCONSISTENT FORMALIZATION OF SET THEORY

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Let T be a formal system containing ZF , whose language include explicitly the arithmetic signature and numerical variables run over ω. Moreover, assume that there are T-formulas of the form $[\psi(\overline{x}), \Delta(Q, \overline{x}, a), a]$, where a is a numerical variable, ψ is a formula of ZF-signature (enriched by numerical variables), Δ a formula of the same signature with a predicate variable Q. Other T-formulas are constructed from ZF-formulas and formulas described above using propositional connectives and quantifiesover individual variables. We consider these formulas as meaningless combinations of symbols.

We assume that T-axiomatics contains the following axiom schemes:

$$[\psi(\overline{x}), \Delta(Q, \overline{x}, a), a]|_0^a \leftrightarrow \psi(\overline{x});$$

$$[\psi(\overline{x}), \Delta(Q, \overline{x}, a), a]|_{b+1}^a \leftrightarrow (\Delta|^{Q,a}_{[\psi(\overline{x}), \Delta(Q, \overline{x}, a), a]|_b^a, b}).$$

Other T-axioms are ZF-axioms ZF-axiom schemes extended (as well as logical schemes) to arbitrary T-formulas. It is not hard to check that T is inconsistent wrt ZF+(*existence of strongly inaccessible cardinals*).

Theorem 0.1 *The system T is ω-inconsistent.*

This fact can be established at metamathematical level. This means that for a suitable T-formula $\varphi(a)$ one can find two metamathematical objects: a formal T-proof of the sentence $\exists a \varphi(a)$ and a primitive recursive function $G(a)$, which construct for a given numeral n a Goedel number of a T-proof of $\neg\varphi(n)$, and this property of the function G can be proved in the recursive arithmetic.

From this fact follows, in particular, that the existence of strongly inaccessible cardinals is refutable in ZF.